Elementary Excitations of Bose Systems

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Introduction

We present two methods to compute the elementary excitations of bosonic systems that rely on the static structure function of the ground state. The roughest one is the Feynman approximation which reproduces qualitatively the real form of the elementary excitation spectrum. The best one uses the correlated basis functions (CBF) approximation which allows interactions between excitations. The last method is used to study superfluid $^4$He and the dilute bose gas in the unitary limit. On one hand we reproduce the well known dynamic structure function of $^4$He, and on the other we show that the spectrum of elementary excitations at unitarity obtained using the metastable ground state in [1] does not contain phonons.

Ground state

The structure factor of the ground state of liquid $^4$He is computed with a Jastrow ansatz using hypernetted chain equations (HNC) [2]. The figure in the right shows HNC/0 results using a MacMillan correlation factor compared to what one gets using the HNC/EL method. The later produces better results specially for low $k$ because:

$$\lim_{k \to 0} S(k) = \frac{\hbar k}{2 \mu c}$$

Superfluid $^4$He

In the Feynman approximation $u_2$ is dropped and the spectrum on the left is obtained. The behaviour is qualitatively correct and we see the phonon, maxon and roton regions. Allowing two-body fluctuations we obtain a richer spectrum which also includes a continuum of excitations and has an elementary excitation curve closer to the experimental one.

CBF approximation

Experiments of scattering with low energy neutrons can be used to determine the dynamic structure function of a system due to the form of the scattering cross-section as [3]

$$\frac{d^3 \sigma}{d \Omega d \omega} = N k^2 b_1 b_2 S(k, \omega)$$

To model $S(k, \omega)$ theoretically using the Correlated Basis Functions approximation (CBF) we attempt to solve the time-dependent Schrödinger equation for a wavefunction of the form

$$\Psi(r, t) = \exp \left( \sum_i u_i(r_i) + \sum_{i<j} u_{ij}(r_i, r_j) \right) \Psi_0(r)$$

Using the convolution and uniform limit approximations one finds [4]

$$S(k, \omega, \epsilon) = -\frac{S(k)}{\pi} \left[ \frac{1}{\hbar \omega - \epsilon(k) - \Sigma(k, \omega)} - \frac{1}{\hbar \omega + \epsilon(k) + \Sigma^*(k, \omega)} \right]$$

where $\Sigma(k, \omega) = \Delta(k, \omega) - i \Gamma(k, \omega)$ is the self-energy of the excitations and reads

$$\Gamma(k, \omega) = \frac{1}{\pi} \int_0^\infty d\epsilon \int_{|k-q|}^{k+q} dp \nu_1(p, q) \nu_2(k, q, p) \rho_1(k, \omega)$$

$$\Delta(k, \omega) = -\frac{1}{\pi} \int_0^\infty d\epsilon \int_{|k-q|}^{k+q} dp \nu_1(p, q) \nu_2(k, q, p) \Gamma(k, \omega)$$

The first integration is not trivial and can be computed using a discretized delta function with adaptive width [5] like the one plotted on the right.

Unitary limit

We reproduce the results of [1] using their correlation factor and the HNC/0 method. The energy as a function of the scattering length is plotted in the right. For small $a$ we reproduce the Bogoliubov mean field prediction [6] and for large $a$ we reach unitarity. The spectrum of elementary excitations does not contain a phononic branch for $k \rightarrow 0$. This can be a property of the unitary regime or just a sign that the trial wave function used in the calculations is not reliable.

Conclusions

The HNC/0 and HNC/EL methods produce reasonably good approximations to the structure factor of the ground state. If the HNC/EL is used in the Feynman and CBF approximations for liquid $^4$He the phonon-roton branch is produced. The CBF produces a phonon-roton curve closer to experimental results but also provides the entire spectrum of excitations that comes from two-body interactions. The unitary wave function used in [1] does not contain a phononic branch at low $k$. It can be a property of unitarity or it can mean that the trial wave function they use must be modified.