ABSTRACT: We want to understand the behaviour of flexoelectricity and surface piezoelectricity and distinguish them in order to go deep into the controversies of the filed. This motivate the construction of a model of continuum flexoelectric theory. The model proposed is a two-dimensional model that integrates the electromechanical equations that include the elastic, dielectric, piezoelectric and flexoelectric effect on a rectangular sample. As the flexoelectric and the surface piezoelectric effects appear on thin films (at the nanoscale) it requires a rich continuum model based on strain gradient elasticity too (we have to consider a second order effect). The high-order partial differential equations of electromechanics can’t be solved with standard finite elements because of the second order effects included. Therefore, method should consists in using a galerkin method with smooth shape functions based on B-Splines. As a first step to understand the surface piezoelectricity we implement the mechanical part of our model, i.e., considering the elastic effect term and the gradient elasticity term. In order to do that we encode from zero the numerical method using MATLAB. We arrive to a very important result about the elastic behaviour of a beam (sample): the size effect of cantilever beam bending. This result, that is known, will validate the development of our flexoelectricity model and the correct realization of the numerical method involved.

### CONTINUUM MODEL OF ELECTROMECHANICAL EQUATIONS (STRONG FORM)

\[ \nabla \cdot \sigma = 0 \quad \nabla \cdot D = 0 \quad \text{where} \\
\sigma_{ij} = \delta_{ij} \varepsilon - \dot{\varepsilon}_{ij,k} \mu_{ij,k} \tilde{E}_{i,k} \\
D_{ij} = \bar{D}_{ij} - \dot{\phi} \varepsilon \delta_{ij} \mu_{ij,k} \tilde{E}_{i,k} \\
\varepsilon = \nabla (u) \\
\phi = -\nabla (\varepsilon) \\
\]

### DISCRETIZED SYSTEM EQUATIONS

Local contribution of each quadrature point to the matrix

\[
\begin{bmatrix}
A_{uu} & A_{up} \\
A_{pu} & A_{pp}
\end{bmatrix}
\begin{bmatrix}
U \\
\phi
\end{bmatrix}
= 
\begin{bmatrix}
f_U \\
f_\phi
\end{bmatrix}
\]

Discrete approximations of our solution

Mechanical displacement and electrical potential

\[
u_{i} = \sum_{k} p_{ik} \bar{u}_{k} \\
\phi_{i} = \sum_{k} p_{ik} \bar{\phi}_{k}
\]

### OUR NUMERICAL METHOD:

- GALERKIN METHOD
- B-SPLINES BASIS FUNCTIONS

### RESULTS

**Size effect of cantilever beam bending**

- `21.5mm`
- `0.29μm`

**Our sample configuration**