Modelling the Cardiovascular System: Pulsating Flow Through a Collapsible Channel
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Summary
The interaction between fluid flows and the deformation of solid structures is important in many engineering applications, as in turbochargers, wind turbines or airplane wings. The flow of blood in the body is also an example involving very complex fluid-structure interactions. Here the flow rate pulsates cyclically, the fluid is non-Newtonian and the precise material properties are difficult to measure accurately. Hence the development of appropriate simplified models and experiments is necessary in order to determine the importance of each of these ingredients and their nature.

Motivation
The introduction of new numerical methods as well as the development of new experimental measuring techniques that produce large amount of data retrieved by biologists, chemists and other scientists has induced a growing interest in interpreting and understanding of many biological systems. These results allow mathematicians, engineers and physicians to verify the correctness of their models, in addition to giving important measurements to describe their behaviour.

Problem
Fluid domain
- Incompressible
- Newtonian
- Time-oscillating inflow pressure \( p_{in} \)

Solid domain
- Massless
- Thickness \( h \)
- Kirchhoff-Love elastic beam (red curve)

Equations

Navier-Stokes Equations
\[
Re \left( \nabla^2 v + (u \cdot \nabla) u \right) = -\nabla P + 2\nabla \times (D(u))
\]

Principle of Virtual Displacements
\[
\int_0^t \left[ (c_0 + \gamma \Delta y + \frac{1}{12} h^2 \kappa \delta \kappa) - \frac{1}{12} h \sqrt{A} \cdot \delta R_w \right] d\tau = 0
\]

Methodology

The Finite Elements Method
Find \( U \in \mathbb{R}^N \) such that
\[
r_j \left( U_1, \ldots, U_N \right) = \int_\Omega R \left( u_y + \sum_{i=1}^N U_i \phi_i \right) \phi_j \ d\Omega = 0, \quad \forall j = 1, \ldots, N.
\]

Oomph-lib
Object-oriented, open-source finite-element library for the simulation of multi-physics problems.

Analysis
- Track the displacement \( dy \) of the midpoint of the elastic membrane.
- Sweep for many values of \( St \) and fix \( Re \), for many different \( Re \).
- Find patterns in order to characterize the response.
- Keep the rest of the parameters (mainly) unchanged

Results

Mean Deviation

Transient Time

Phase Shift

Amplitude

Conclusions
- The systems behaves as a Driven Damped Oscillator.
- The resonant effects appear at out of normality values of the parameters.

Future Work
- Find an analytic relation between the model parameters and those of the DDO.
- Differences from the results expected. Different approach.